Student Notes

Student Study Session Topic: Important Theorems

Facts, “truth,” ideas, etc. in mathematics are known as *definitions*, *theorems,* and *postulates* (also known as *axioms or assumptions*). Theorems require proof based on other theorems, definitions, or postulates. Definition name things and do not require proof. Postulates are where things start and are assumed to be true and do not therefore require proof. We will not be concerned with proof today.

Theorems and postulates have the form

IF (some things are true), **then** (some other thing is true).

For example, the statement “**If** a function is differentiable at a point, **then** it is continuous at that point,” is an important theorem in calculus. The “if” part is called the *hypothesis* and the “then” part is called the *conclusion*. Theorems are also called conditional statements.

There are three statements related to each theorem.

- The *converse* is formed by interchanging the hypothesis and conclusion. The converse of a true theorem may be true, but often is not. The converse of the theorem above is “If a function is continuous at a point, then it is differentiable at the point” – this statement is false; consider the *counterexample*: $y = |x|$.

- The *inverse* is formed by negating the hypothesis and conclusion of the theorem Example: “If a function is not differentiable at a point, then it is not continuous at a point.” This statement is also false.

- The *contrapositive* is formed by negating and interchanging the hypothesis and conclusion of a theorem. Example: “If a function is not continuous at a point, then it is not differentiable at that point.” This is true.

A theorem and its contrapositive are either both true or both false; a converse and an inverse are either both true or both false. However, a theorem and its converse do not have to be both true or both false, although they may be. Thus, if the contrapositive is true, the theorem must be true also.

A *definition* names something and is a biconditional statement. Its form is

(something is true) **if, and only, if** (something else is true).

Example: A quadrilateral is a square if and only if it has 4 congruent sides and 4 right angles. This same definition could be written “A quadrilateral has 4 congruent sides and 4 right angles if and only if it is a square. Definitions are always reversible; their converse is always true. This is why they are called biconditional statements.

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Learning Theorems

Theorems should be understood more than memorized. Understand their structure and learn to "play" with them. Learn their meaning by looking at them *graphically*, *numerically*, *analytically* and *verbally*.

To "play" with a theorem: The assumptions matter. Each hypothesis is necessary for some reason; understand why each is there. Change one of the hypotheses and see what happens and what changes. What would be different if this were not here?

Check the *converse*, *inverse*, and *contrapositive*.

Learn about *counterexamples*.

The words "any", "every" and "all" are used in many theorems and definitions. The three words are always interchangeable: read the theorem and replace one of these words with each of the others. This may help you better understand what the theorem or definition means.

- For *all* real numbers x, $x^2 \geq 0$
- For *any* real number x, $x^2 \geq 0$
- For *every* real number x, $x^2 \geq 0$

Proofs are, believe it or not, less important to understanding the theorem. If you're convinced a theorem is true, or can see why it's true from the diagram, then you can scan or even skip the proof.

Understanding a theorem is different than understanding its proof. A reason to carefully study proofs is to learn how to do proofs yourself. Another reason is to show you why the theorem is true. Also, some proofs help you understand the theorem better (some do not).

The main theorems tested on the AP Calculus exams are listed. There may be others.

Intermediate value theorem:

If f is continuous on a closed interval [a, b] and $f(a) \neq f(b)$, then for every value of M between $f(a)$ and $f(b)$, there exist at least one value of c in the open interval (a, b), such that $f(c) = M$

In other words, a continuous function takes on all the values between any two of its values.

Extreme value theorem

If f is continuous on a closed interval [a, b], then f takes on a maximum and a minimum value on that interval.

A more formal wording asserts that there is a value in the domain for which the function assumes a value greater than or equal all its other values and also less than or equal to all its other values:

If f is continuous on the closed interval $[a, b]$, then:

1. there exists a number c_1 in $[a, b]$ such that $f(x) \le f(c_1)$ for all x in $[a, b]$
2. there exists a number c_2 in $[a, b]$ such that $f(x) \ge f(c_2)$ for all x in $[a, b]$

Notes: (1) A function may attain its maximum and minimum value more than once. For example, the maximum value of $y = \sin(x)$ is 1 and it reaches this value many, many times; (2) the extreme values often occur at the endpoint of the domain, and (3) for a constant function the maximum and minimum values are equal (in fact all the values are equal).

Mean value theorem and Rolle's Theorem

Rolle: If If f is continuous on the closed interval $[a, b]$, and differentiable on the open interval (a, b), and $f(a) = f(b)$, then there exists a number c in the open interval (a, b) such that $f'(c) = 0$.

Graphically this means that somewhere between the places where a function takes on the same value there must be a place with a horizontal tangent (and therefore a relative maximum or minimum).

MVT: If f is continuous on the closed interval $[a, b]$, and differentiable on the open interval (a, b), then there exists a number c in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Graphically this means that somewhere in the interval the tangent line must be parallel to the line between the endpoints.

Increasing/Decreasing theorem

If f is continuous on the closed interval $[a, b]$, and differentiable on the open interval (a, b), and if for all c in the open interval (a, b), $f'(c) > 0$ ($f'(c) < 0$), then f is increasing (decreasing) on the closed interval $[a, b]$.

Note: If $f'(c) = 0$, this theorem does not apply. Why?

First derivative test

If f is differentiable and c is a critical point of f and if $f'(x)$ changes from positive to negative at x = c then $f(c)$ is a local maximum of f.

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If *f* is differentiable and c is a critical point of *f* and if $f'(x)$ changes from negative to positive at $x = c$ then $f(c)$ is a local minimum of *f*.

Second derivative test

Let *f* be a function such that $f'(c)=0$ and the second derivative of *f* exists on an open interval containing *c*, then if $f''(c)>0$, *f* has a local *minimum* value at $x = c$.

Let *f* be a function such that $f'(c)=0$ and the second derivative of *f* exists on an open interval containing *c*, then if $f''(c)<0$, *f* has a local *maximum* value at $x = c$.

If $f''(c)=0$, the second derivative test cannot be used. The function may have a maximum, a minimum or neither at *c*.

Differentiability implies Continuity

If a function is differentiable at a point, then it is continuous at the point.

Note: The converse is *false*: continuous functions are not necessarily differentiable.

The Fundamental Theorem of Calculus

If $F(x)=\int_a^x f(t)\,dt$, then $F'(x)=f(x)$

If $F(x)=\int_a^{g(x)} f(t)\,dx$, then $F'(x)=f(g(x))g'(x)$

If $F'(x)=f(x)$, then $\int_a^b f(x)\,dx = F(b)-F(a)$

$$f(x)=f(a)+\int_a^x f'(t)\,dt$$

1. Work these free-response question parts as directed by your teacher:
    a. 2006 (Form B) AB 6 (c) and (d).
    b. 1999 AB 3 – BC 3 (b)
    c. 2005 AB 3 (d)
    d. 2008 BC 5 (a) (b)
    e. 2005 AB 5 (b) (d)

2. Next go on to the multiple-choice questions (Calculators for 6 – 9 only. 11 is a BC only question).

WATCH and LISTEN to the multiple-choice questions being solved

Go to http://tinyurl.com/NMSI-Math-7 Click on the "Full Screen" arrow. Then click anywhere on the page to see and hear from that point on.

Click anywhere to go back anytime.

Multiple-choice answers: 1 A, 2 E, 3 B, 4 B, 5 E, 6 C, 7 C, 8 D, 9 D, 10 E, 11 C.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

6. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

   (a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

   (b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

   (c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

   (d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

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(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

(a) $\int_{30}^{60} |v(t)|\, dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)|\, dt$:

$$A = \frac{1}{2}(14 + 10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

$2: \begin{cases} 1: \text{explanation} \\ 1: \text{value} \end{cases}$

(b) $\int_{0}^{30} a(t)\, dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\int_{0}^{30} a(t)\, dt = \int_{0}^{30} v'(t)\, dt = v(30) - v(0)$$
$$= -14 - (-20) = 6 \text{ ft/sec}$$

$2: \begin{cases} 1: \text{explanation} \\ 1: \text{value} \end{cases}$

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

$2: \begin{cases} 1: v(35) < -5 < v(50) \\ 1: \text{Yes; refers to IVT or hypotheses} \end{cases}$

(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

$2: \begin{cases} 1: v(0) = v(25) \\ 1: \text{Yes; refers to MVT or hypotheses} \end{cases}$

Units of ft in (a) and ft/sec in (b)

1 : units in (a) and (b)

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$. Using correct units, explain the meaning of your answer in terms of water flow.

   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

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3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

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3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

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(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

(c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

(a) $\int_0^{24} R(t)\,dt \approx 6[R(3) + R(9) + R(15) + R(21)]$

$= 6[10.4 + 11.2 + 11.3 + 10.2]$

$= 258.6$ gallons

This is an approximation to the total flow in gallons of water from the pipe in the 24-hour period.

$3\begin{cases} 1\text{: } R(3) + R(9) + R(15) + R(21) \\ 1\text{: answer} \\ 1\text{: explanation} \end{cases}$

(b) Yes;

Since $R(0) = R(24) = 9.6$, the Mean Value Theorem guarantees that there is a t, $0 < t < 24$, such that $R'(t) = 0$.

$2\begin{cases} 1\text{: answer} \\ 1\text{: MVT or equivalent} \end{cases}$

(c) Average rate of flow

$\approx$ average value of $Q(t)$

$$= \frac{1}{24}\int_0^{24} \frac{1}{79}(768 + 23t - t^2)\,dt$$

$= 10.785$ gal/hr or 10.784 gal/hr

$3\begin{cases} 1\text{: limits and average value constant} \\ 1\text{: } Q(t) \text{ as integrand} \\ 1\text{: answer} \end{cases}$

(units) Gallons in part (a) and gallons/hr in part (c), or equivalent.

1: units

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

   (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

   (c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

   (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

WRITE ALL WORK IN THE TEST BOOKLET.

END OF PART A OF SECTION II

Question 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

(a) $\dfrac{T(8)-T(6)}{8-6} = \dfrac{55-62}{2} = -\dfrac{7}{2}$ °C/cm

1 : answer

(b) $\dfrac{1}{8}\int_0^8 T(x)\,dx$

Trapezoidal approximation for $\int_0^8 T(x)\,dx$:

$$A = \frac{100+93}{2}\cdot 1 + \frac{93+70}{2}\cdot 4 + \frac{70+62}{2}\cdot 1 + \frac{62+55}{2}\cdot 2$$

Average temperature $\approx \dfrac{1}{8}A = 75.6875$°C

$3 : \begin{cases} 1 : \frac{1}{8}\int_0^8 T(x)\,dx \\ 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^8 T'(x)\,dx = T(8) - T(0) = 55 - 100 = -45$°C

The temperature drops 45°C from the heated end of the wire to the other end of the wire.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$

(d) Average rate of change of temperature on $[1, 5]$ is $\dfrac{70-93}{5-1} = -5.75$.

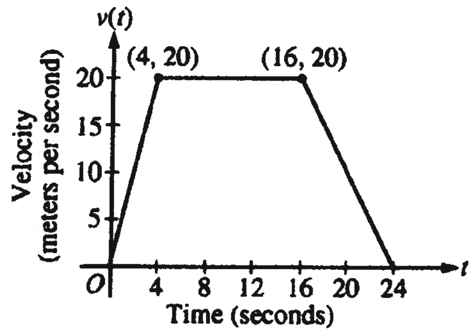
Average rate of change of temperature on $[5, 6]$ is $\dfrac{62-70}{6-5} = -8$.

No. By the MVT, $T'(c_1) = -5.75$ for some c_1 in the interval $(1, 5)$ and $T'(c_2) = -8$ for some c_2 in the interval $(5, 6)$. It follows that T' must decrease somewhere in the interval (c_1, c_2). Therefore T'' is not positive for every x in $[0, 8]$.

$2 : \begin{cases} 1 : \text{two slopes of secant lines} \\ 1 : \text{answer with explanation} \end{cases}$

Units of °C/cm in (a), and °C in (b) and (c)

1 : units in (a), (b), and (c)



5. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

WRITE ALL WORK IN THE TEST BOOKLET.

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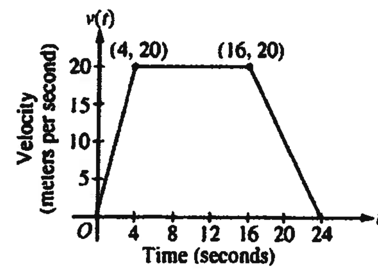
Question 5



A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(a) $\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$

The car travels 360 meters in these 24 seconds.

$2: \begin{cases} 1: \text{value} \\ 1: \text{meaning with units} \end{cases}$

(b) $v'(4)$ does not exist because

$$\lim_{t \to 4^-} \left(\frac{v(t) - v(4)}{t - 4} \right) = 5 \ne 0 = \lim_{t \to 4^+} \left(\frac{v(t) - v(4)}{t - 4} \right).$$

$$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

$3: \begin{cases} 1: v'(4) \text{ does not exist, with explanation} \\ 1: v'(20) \\ 1: \text{units} \end{cases}$

(c)

$$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$2: \begin{cases} 1: \text{finds the values } 5, 0, -\frac{5}{2} \\ 1: \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of v on $[8, 20]$ is

$$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2.$$

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

$2: \begin{cases} 1: \text{average rate of change of } v \text{ on } [8, 20] \\ 1: \text{answer with explanation} \end{cases}$

Question 5

The derivative of a function f is given by $f'(x) = (x-3)e^x$ for $x > 0$, and $f(1) = 7$.

(a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

(b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

(c) Find the value of $f(3)$.

(a) $f'(x) < 0$ for $0 < x < 3$ and $f'(x) > 0$ for $x > 3$

Therefore, f has a relative minimum at $x = 3$.

$$2 : \begin{cases} 1\text{: minimum at } x = 3 \\ 1\text{: justification} \end{cases}$$

(b) $f''(x) = e^x + (x-3)e^x = (x-2)e^x$

$f''(x) > 0$ for $x > 2$

$f'(x) < 0$ for $0 < x < 3$

Therefore, the graph of f is both decreasing and concave up on the interval $2 < x < 3$.

$$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer with reason} \end{cases}$$

(c) $f(3) = f(1) + \int_1^3 f'(x)\,dx = 7 + \int_1^3 (x-3)e^x\,dx$

$$u = x - 3 \quad dv = e^x\,dx$$

$$du = dx \qquad v = e^x$$

$$f(3) = 7 + (x-3)e^x\Big|_1^3 - \int_1^3 e^x\,dx$$

$$= 7 + \left((x-3)e^x - e^x\right)\Big|_1^3$$

$$= 7 + 3e - e^3$$

$$4: \begin{cases} 1 : \text{uses initial condition} \\ 2 : \text{integration by parts} \\ 1 : \text{answer} \end{cases}$$

Name: ________________________ Class: ___________________ Date: __________ ID: A

Teaching Theorems Multiple-choice Examlpes

Multiple Choice
Identify the choice that best completes the statement or answers the question.

x	0	1	2
$f(x)$	1	k	2

____ 1.

The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if k=

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. 3

____ 2. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c)=3$.

a. None
b. I only
c. I and II only
d. I and III only
e. I, II and III

____ 3. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) =-5$ and $f(1) = 4$, which of the following statements could be false?

a. There exists c, where $-2 < c < 1$, such that $f(c) = 0$.
b. There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.
c. There exists c, where $-2 < c < 1$, such that $f(c) =3$.
d. There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.
e. There exists c, where $-2 \le c \le 1$ such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

Name: ________________________ **ID: A**

____ 4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

a. $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.
b. $f'(c) = 0$ for come c such that $a < c < b$.
c. f has a minimum value on $a \le x \le b$.
d. f has a maximum value on $a \le x \le b$.
e. $\int_a^b f(x)dx$ exists.

____ 5.

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

The function f is continuous and differentiable on the closed interval [0,4]. The table above gives selected values of f on this interval. Which of the following statements must be true?

a. The minimum value of f on [0, 4] is 2.
b. The maximum value of f on [0, 4] is 4.
c. $f(x) > 0$ for $0 < x < 4$
d. $f'(x) < 0$ for $2 < x < 4$
e. There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

____ 6. Let $f(x) = \int_0^{x^2} \sin t\,dt$. At how many points in the closed interval $[0, \sqrt{\pi}]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

a. Zero
b. One
c. Two
d. Three
e. Four

____ 7. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1,4]?

a. 0.456
b. 1.244
c. 2.164
d. 2.342
e. 2.452

____ 8. If f is the antiderivative of $\dfrac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

a. -0.012
b. 0
c. 0.016
d. 0.376
e. 0.629

____ 9. If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$

a. -3
b. -2
c. 2
d. 3
e. 18

____ 10. $\dfrac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)dt\right) =$

a. $-\cos\left(x^6\right)$
b. $\sin\left(x^3\right)$
c. $\sin\left(x^6\right)$
d. $2x\sin\left(x^3\right)$
e. $2x\sin\left(x^6\right)$

____ 11. $\lim\limits_{x\to 1} \dfrac{\int_1^x e^{t^2}dt}{x^2-1}$ is

a. 0
b. 1
c. $\dfrac{e}{2}$
d. e
e. nonexistent